Detectability of several ideal spatial patterns

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The relative detectability of three potentially optimal visual signals derived on the basis of statistical decision theory was measured and compared with that of a one-dimensional analog of the most detectable signal found by Watson *et al.* [Invest. Ophthalmol. Vis. Sci. Suppl. **20,** 178 (1981)]. The latter signal was marginally easier to detect than the other three, but the small differences in detectability preclude our making strong inferences about the nature of visual pattern detection.

INTRODUCTION

The specification of an optimal linear device for detecting a signal of known characteristics forms one of the classic problems of statistical decision theory.[1–4] The classic problem has a simple obverse in the specification of an optimal signal to be detected by a device whose characteristics are known. Recent experiments in spatial vision[5,6] appear to provide enough detailed information about the (one-dimensional) spatial-frequency tuning of channels in the visual system to enable us to test our knowledge of the system by deriving an optimal pattern for detection and comparing its detectability with that of other highly detectable signals.[7] We have restricted our investigation to stimuli of one spatial dimension because our information about the underlying process comes from experiments that used a similarly restricted class of stimuli. Extension of the argument to two- (or more-) dimensional stimuli will be straightforward once sufficient data become available.

The channel through which a low-contrast signal is detected may be specified either by its spatial weighting function $h(x)$ or, equivalently, by the transfer function $H(f)$, which is the Fourier-integral transform of the spatial weighting function.[8] The output of such a system in response to an arbitrary one-dimensional stimulus is given by the convolution of the stimulus and the spatial weighting function.

We shall assume that the detection of a given pattern through a channel is determined by the square of the output of the channel Ωs and that detection performance is limited by some form of broadband internal noise.[9]

Since the characteristics of the channel (in particular, its spatial weighting function) are assumed to be fixed, our problem reduces to that of finding a stimulus that maximizes Ωs. It is readily shown by means of a Schwarz inequality[3,10] that one such stimulus is simple $h(-x)$. Thus, once $h(x)$ is known, the optimal signal is determined.

Although there have been some attempts to measure spatial weighting functions directly,[11–13] it is usually easier to specify the properties of a channel by its transfer function $H(f)$, given by

$$H(f) = \int_{-\infty}^{\infty} h(x)\exp(-j2\pi fx)\mathrm{d}x, \qquad (1)$$

where f is spatial frequency in cycles per degree (c/deg) of visual angle. The complex function $H(f)$ is most conveniently written as

$$H(f) = |H(f)|\exp[j\phi(x)], \qquad (2)$$

where $|H(f)|$ is the attenuation characteristic of the channel and $\phi(f)$ is its phase characteristic.

Henning *et al.*[6] calculated the attenuation characteristic of channels $|H(f)|$ from masking experiments; the form of $|H(f)|$ that they derive is linear on double logarithmic coordinates and has the same slopes at different spatial frequencies. Thus, in order to determine $h(x)$, and hence the optimal signal for one of these channels, it is necessary only to associate a plausible phase spectrum with $|H(f)|$ and to calculate $h(-x)$ by the appropriate inverse transform.

We have tried three phase spectra: that associated with a channel whose spatial weighting function is (1) even symmetric (ESYM) or (2) odd symmetric (OSYM) about its center and (3) one that has a phase characteristic that is the Hilbert transform of $|H(f)|$ (MINPH). These three phase conditions are not exhaustive. Indeed, there are an infinite number of other possible phase characteristics that might be tried. However, the OSYM and ESYM characteristics are plausible on physiological evidence, and the third is a compromise between those two. All three signals were designed for a channel centered on 7 c/deg because Watson *et al.*[7] found this to be the center frequency of their most sensitive channel. Our observers may differ slightly in this respect from those of

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Watson *et al.*, but that should not affect the comparisons that we make among our signals, all of which are centered on 7 c/deg.

In addition to the three potentially optimal signals described above, we used a 7 c/deg grating whose extent was modulated by a Gaussian envelope that fell to $1/e$ of its peak amplitude in 1.5 cycles of the grating of WBR. This is a one-dimensional version of the grating used by Watson *et al.*

METHODS

Thresholds for the four patterns were measured concurrently by using a staircase procedure in a temporal two-alternative forced-choice task. Eight staircases (two for each stimulus) were randomly interleaved during each session.

Stimuli. Signal waveforms were luminance patterns varying in one spatial dimension presented on an oscilloscope screen (Hewlett-Packard HP 1332a, P-31 phosphor) using the technique of Schade.[14] The space average luminance was 45 cd m^{-2} and was unaltered by the presentation of the stimuli. The display subtended 3.0 deg by 3.0 deg at the viewing distance of 1.7 m and had a surrounding square 14 deg by 14 deg, which was matched to the color and brightness of the display.

The stimuli were digital approximations to the waveforms described in the introduction, created by sampling the required waveform at intervals of 0.0093 deg and displaying the sampled waveform by means of a 12-bit digital-to-analog converter (DAC). Our three potentially ideal stimuli had the same amplitude spectrum,[6] which had a slope of 0.267 log unit per halving of spatial frequency below 7 c/deg and a slope of 0.159 log unit per doubling of spatial frequency above 7 c/deg, but they differed in their phase spectra. The fourth stimulus was the windowed sinusoidal grating of Watson *et al.* All stimuli had the same Gaussian temporal waveform with a time constant of 80 msec derived from a second 12-bit DAC, the output of which multiplied the spatial waveform. The product was low-pass filtered and led to the Z axis of the oscilloscope through a programmable attenuator. The spatial profiles of the four stimuli, matched for peak luminance in this case, are shown in Fig. 1. Although Fig. 1 is convenient for comparing the shapes of the signals, it should be borne in mind that relative detectability is measured for signals equated with respect to the square of the signal contrast integrated over the extent of the signal. Luminances and contrasts were calibrated using a photometer (UDT 80X).

Procedure. On each trial the stimulus was presented in one of two observation intervals, each signaled by a tone. The observer's task was to indicate, by pressing one of two switches, which interval contained the stimulus. A second press initiated the next trial.

Each stimulus was clearly visible at the start of each session, and its contrast was adjusted during the session in accordance with the observer's performance. Three consecutive correct responses to a stimulus caused a reduction in its contrast on the next trial on which it was presented. An incorrect response caused an increase. The contrast changed in steps of 0.3 log unit at the start of each session, but the first three times a contrast step was changed in sign [that is, when the observer changed from responding correctly to responding incorrectly (or vice versa)], the absolute size of the step was halved.

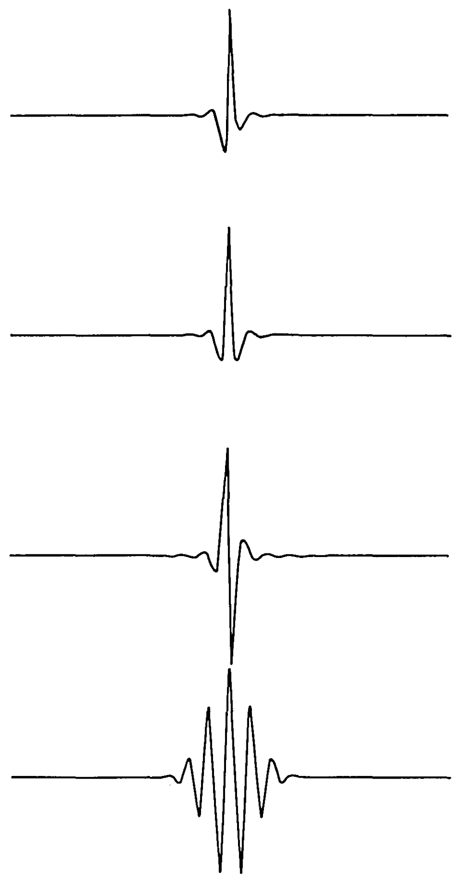

Fig. 1. Spatial luminance profiles of the four stimuli showing luminance as a function of distance. Stimuli MINPH, OSYM, and ESYM are optimal signals derived from the treatment of Henning *et al.*[6] of the data of Stromeyer and Julesz.[5] The three stimuli have identically shaped amplitude spectra but phase spectra that correspond to the Hilbert transform of $H(f)$ (MINPH), odd symmetric (OSYM), and even symmetric (ESYM) luminance profiles. Stimulus WBR is the product of a 7-c/deg sinusoid and a Gaussian envelope with a space constant of 3/14 deg.

RESULTS

The detectability of each stimulus can be expressed as the ratio of the contrast at which the stimulus energy—the square of the signal contrast integrated over the extent of the display—is unity to the contrast at threshold. The most detectable stimulus is then the one that has the highest ratio. The detectability of each of our four stimuli for each of our three observers is plotted in Fig. 2.

The results show two important features. First, all three observers found the Gaussian-windowed grating of Watson *et al.* to be the most detectable stimulus. Second, and more important, the differences in detectability among stimuli are small in comparison with the variation among individual estimates of detectability and with the differences in sensitivity among observers.

DISCUSSION

There are several important assumptions implicit in our derivation of an optimal signal. First, we have assumed our channels to be linear and shift invariant. The assumption of linearity is probably not unreasonable for the low-contrast signals that we use; even if the visual system were highly nonlinear we might reasonably expect to have the advantage of small-signal linearity.[15] The assumption of homogeneity, on the other hand, is unlikely to be true,[13,16] and we are unable to address the issue directly with our experiment. However, the channel characteristics that we use are derived from experiments using relatively large visual fields. Consequently it is reasonable to assume that any effects of inhomogeneity are incorporated into our spatial weighting functions.

A second and related difficulty is the open question of the number of different types of channel and the extent to which the responses of each type to our stimuli might influence their detectability. Wilson and Bergen[13] argue for only four types of channel. Only two of these have sufficient sensitivity to respond significantly to our low-contrast stimuli. Watson and Robson[17] and Watson,[18] however, argue for a minimum of seven types spanning the range of spatial frequencies to which the visual system responds. Pollen and Feldon[19] argue on physiological grounds for about nine, partitioning each group[20] into a class with even symmetric and a class with odd symmetric spatial weighting functions. Again we may assume that all but the last of these factors had their effects incorporated into the spatial weighting function (or equivalently, the transfer function) that we use to determine the ideal signal.

The notion of there being both odd symmetric and even symmetric spatial weighting functions is certainly not implausible on the basis of our results; there is little difference in detectability between our OSYM and ESYM signals, and

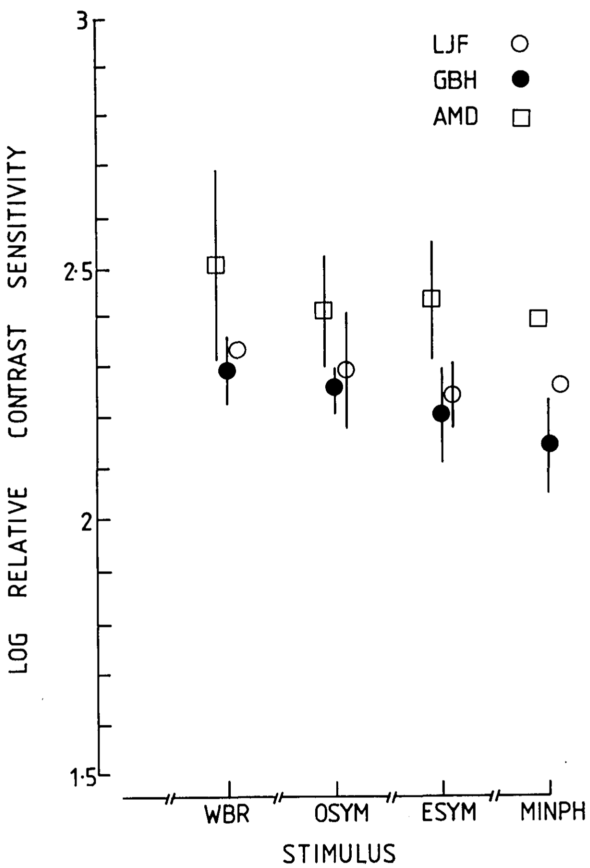

Fig. 2. The detectability of the four stimuli for three observers. Vertical bars extend one standard error above and below the center of each symbol. AMD and GBH are the authors.

this might reasonably be interpreted as evidence for equally sensitive systems of odd and even symmetry.

It might also be argued that an even symmetric channel centered on the peak of the OSYM signal, rather than on its center, might be almost as sensitive to that signal as an odd symmetric channel. An OSYM signal detected through a channel with a symmetric spatial weighting function centered on the bright peak of the asymmetric signal should be 0.174 log unit (a factor of 1.5) less detectable than the ESYM signal. However, the two signals are equally detectable. Thus our data support the notion of two mechanisms, one with even symmetric and one with odd symmetric spatial weighting functions.

The same technique allows us to predict the detectability of our approximation of the most detectable signal found by Watson *et al.*[7] This signal, which is even symmetric, should be 1.09 log units less detectable than the optimal signal for our even symmetric channel and, at best, 1.11 log units less detectable than the optimal signal for our odd symmetric channel. Our results, then, based as they must be on the behavior of the channel (or channels) most sensitive to the stimulus presented, support neither of these predictions.

Finally, it is clear that there are only small differences in the detectability of any of the signals that we have used. The small difference occurs in spite of not inconsiderable differences among the signals themselves. Now treating each different signal as if it were an optimal signal (and hence determining the characteristics of the channel for which it is optimal) predicts channel shapes that also differ significantly in terms of both their attenuation characteristics and their phase responses. Since large changes in these characteristics reflect only small changes in detectability, it is clear that there is little to be gained from the quest for optimal signals. It may even be that we are dealing with a system that optimizes the detectability of a class of signals[21] or indeed with one that adjusts its characteristics to optimize its performance for whatever signal it is required to detect.

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